

1. Obtén las soluciones del siguiente sistema de ecuaciones:

$$\begin{cases} y^2 - x^2 = -3 \\ xy = -2 \end{cases}$$

$$\boxed{x = \frac{-2}{y}}; \quad y^2 - \left(\frac{-2}{y}\right)^2 = -3; \quad y^2 - \frac{4}{y^2} = -3;$$

$$\frac{y^4 - 4}{y^2} = -3; \quad y^4 - 4 = -3y^2; \quad y^4 + 3y^2 - 4 = 0; \quad \boxed{y^2 = z}$$

$$z^2 + 3z - 4 = 0; \quad z = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} = \begin{cases} 1 & \text{se sirve} \\ -4 & \text{no sirve} \end{cases}$$

$$y^2 = 1; \quad y = \pm\sqrt{1} = \pm 1; \quad \boxed{y=1; x=-2}; \quad \boxed{y=-1; x=2}$$

2. Halla las soluciones del sistema:

$$\begin{cases} x - y = 9 \\ \log x - \log y = 1 \end{cases}$$

$$\boxed{x = 9 + y}; \quad \log(9 + y) - \log y = 1; \quad \log \frac{9+y}{y} = 1;$$

$$\frac{9+y}{y} = 10; \quad 9+y = 10y; \quad 9y = 9; \quad \boxed{y=1; x=10}$$

3. La suma de dos números es 3 y la suma de sus inversos es $-\frac{3}{4}$. ¿De qué números se trata?

$$\begin{cases} x + y = 3 \\ \frac{1}{x} + \frac{1}{y} = -\frac{3}{4} \end{cases} \quad \boxed{x = 3 - y}$$

$$\frac{1}{3-y} + \frac{1}{y} = -\frac{3}{4}; \quad \frac{4y}{(3-y) \cdot y \cdot 4} + \frac{(3-y) \cdot 4}{(3-y) \cdot y \cdot 4} = -\frac{3(3-y)y}{4}$$

$$4y + 12 - 4y = -9y + 3y^2; \quad 3y^2 - 9y - 12 = 0;$$

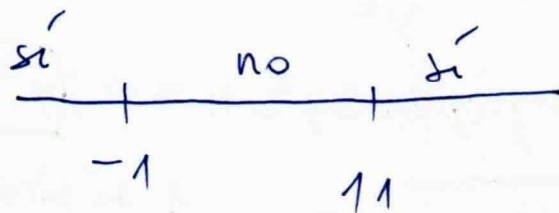
$$y^2 - 3y - 4 = 0 \quad y = \frac{3 \pm \sqrt{(-3)^2 - 4(-4)}}{2} = \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2} = \begin{cases} 4 \\ -1 \end{cases}$$

$$\boxed{x = 3 - 4 = -1; y = 4}$$

$$\boxed{x = 3 - (-1) = 4; y = -1}$$

4. Resuelve la siguiente inecuación: $\left| \frac{x-5}{2} \right| > 3$

$$\frac{x-5}{2} = \pm 3 ; x-5 = \pm 6 ; x = 5 \pm 6 \quad \begin{matrix} 11 \\ -1 \end{matrix}$$



$$(-\infty, -1) \cup (11, +\infty)$$

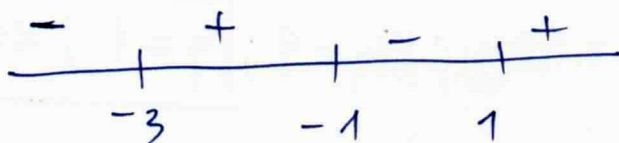
$$x = -2 \quad \left| \frac{-2-5}{2} \right| = \frac{7}{2} > 3 \text{ sí}$$

$$\left| \frac{0-5}{2} \right| = \frac{5}{2} > 3 \text{ no}$$

$$\left| \frac{12-5}{2} \right| = \frac{7}{2} > 3 \text{ sí}$$

5. Resolver la inecuación: $\frac{x^2-1}{(x+3)} \geq 0$

$$\frac{x^2-1}{x+3} = 0 ; \quad \begin{matrix} x^2-1=0 ; x=\pm 1 \\ x+3=0 ; x=-3 \end{matrix}$$



$$x = -4 ; \quad \frac{(-4)^2-1}{-4+3} = \frac{15}{-1} = -15 < 0$$

$$x = -2 ; \quad \frac{(-2)^2-1}{-2+3} = \frac{3}{1} = 3 > 0$$

$$x = 0 ; \quad \frac{0^2-1}{0+3} = \frac{-1}{3} < 0 ; \quad x = 2 ; \quad \frac{2^2-1}{2+3} = \frac{3}{5} > 0$$

$$[-3, -1] \cup [1, +\infty)$$

6. Resuelve el siguiente sistema: $\begin{cases} 2x-y \leq 1 \\ x+4y \geq 2 \end{cases}$

$$2x - y = 1 \quad | \quad x + 4y = 2$$

x	y
0	-1
2	3

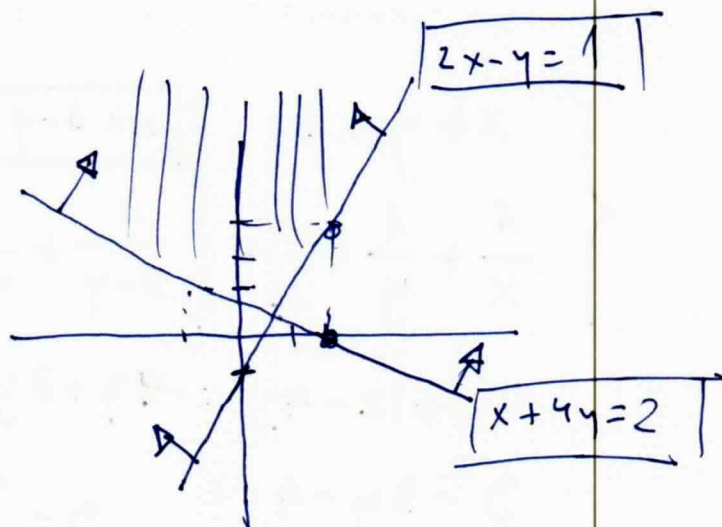
$$x=0 ; y=0$$

$$2 \cdot 0 - 0 = 0 \leq 1 \text{ sí}$$

x	y
2	0
-2	1

$$x=0 ; y=0$$

$$0 + 4 \cdot 0 = 0 < 2 \text{ no}$$



7. Resolver el sistema: $\begin{cases} x + y = 18 \\ xy = y + 6x + 4 \end{cases}$

$$\boxed{x = 18 - y} \quad (18 - y)y = y + 6(18 - y) + 4;$$

$$18y - y^2 = y + 108 - 6y + 4; -y^2 + 23y - 112 = 0$$

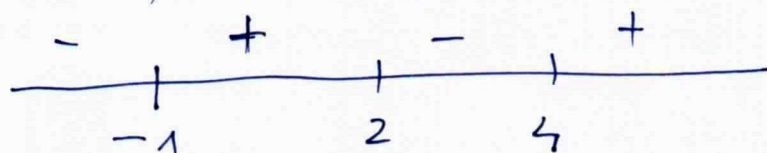
$$y = \frac{-23 \pm \sqrt{23^2 - 4(-1)(-112)}}{-2} = \frac{-23 \pm \sqrt{81}}{-2} = \frac{-23 \pm 9}{-2} = \begin{cases} 7 \\ 16 \end{cases}$$

$$x = 18 - 7 = 11; y = 7; \boxed{x = 11; y = 7}$$

$$x = 18 - 16 = 2; y = 16; \boxed{x = 2; y = 16}$$

8. Resolver la inecuación siguiente: $\frac{(x+1) \cdot (x-2)}{x-4} < 0$

$$x+1=0; x-2=0; x-4=0 \rightarrow x=-1; x=2; x=4$$



$$x = -2 \quad \frac{(-2+1)(-2-2)}{-2-4} = \frac{-1 \cdot (-4)}{-6} = \frac{4}{-6} < 0$$

$$x = 0 \quad \frac{(0+1)(0-2)}{0-4} = \frac{1 \cdot (-2)}{-4} = \frac{-2}{-4} = \frac{1}{2} > 0$$

$$x = 3 \quad \frac{(3+1)(3-2)}{3-4} = \frac{4 \cdot 1}{-1} = \frac{4}{-1} = -4 < 0$$

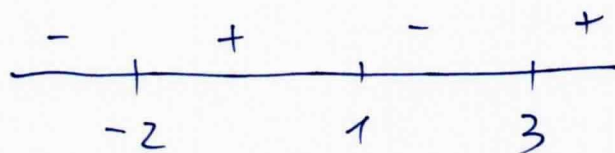
$$\begin{aligned} x &= 5 \\ \frac{(5+1)(5-2)}{5-4} &= \frac{6 \cdot 3}{1} = 18 > 0 \\ \hline &(-\infty, -1) \cup (2, 4) \end{aligned}$$

9. Resolver la siguiente inecuación: $x^3 - 2x^2 - 5x + 6 \leq 0$

$$x^3 - 2x^2 - 5x + 6 = 0$$

	1	-2	-5	6
1		1	-1	-6
	1	-1	-6	0
3		3	6	
	1	2	0	
-2		-2		
	1	0		

$$(-\infty, -2] \cup [1, 3]$$



$$x = -3; (-3)^3 - 2 \cdot (-3)^2 - 5(-3) + 6 = -27 - 18 + 15 + 6 < 0$$

$$x = 0; 0^3 - 2 \cdot 0^2 - 5 \cdot 0 + 6 = 6 > 0$$

$$x = 2; 2^3 - 2 \cdot 2^2 - 5 \cdot 2 + 6 = 8 - 8 - 10 + 6 = -4 < 0$$

$$x = 4; 4^3 - 2 \cdot 4^2 - 5 \cdot 4 + 6 = 18 > 0$$